JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 15, No. 5, September–October 1992

Detection and Identification of Axial Flow Compressor Instabilities

G. Vachtsevanos,* H. Kang,† J. Cheng,‡ and I. Kim‡
Georgia Institute of Technology, Atlanta, Georgia 30332

A new approach to failure detection and identification is proposed that combines an analytic estimation method and an intelligent identification scheme in such a way that sensitivity to true failure modes is enhanced while the possibility of false alarms is reduced. We employ a real-time recursive parameter estimation algorithm with covariance resetting that triggers the fault detection and identification routine only when potential failure modes are anticipated. A possibilistic scheme based on fuzzy set theory is applied to the identification part of the algorithm with computational efficiency. At the final stage of the algorithm, an index is computed—the degree of certainty—based on Dempster-Shafer theory, which measures the reliability of the decision. The proposed algorithm has been applied successfully to the detection of rotating stall and surge instabilities in axial flow compressors.

I. Introduction

RECENTLY, some strategic issues and approaches to failure detection and identification (FDI) have been addressed by several investigators. The first issue is the performance of FDI so that detection delays and false alarms may be avoided.[1] Second, a failure model should reflect a finite number of failure modes that are anticipated (predictable). Third, the design is faced with the tradeoffs of hardware redundancy and software complexity from an implementation point of view.[2,3] Fourth, failure detectability and identifiability can be described in terms of sensitivity and distinguishability of the failure modes. Finally, robustness of FDI in the presence of modeling errors adds more significance to the modeling point of view.

Most of available FDI strategies are related to the multiple model (MM) approach in which innovation-based systems or detection estimation methods are employed. The MM scheme can use existing Kalman filters without any changes and a wide range of statistical test procedures such as the generalized likelihood ratio (GLR) test[2] or the sequential probability ratio (SPR) test[4] that can be applied to additive failures or event-driven faults. For linear systems, the failure sensitive filter approach provides a solution to FDI. The Beard-Jones detection filters (BJDF)[5] or the Luenberger observer filters (LOF)[6] may be applied to detect a wide variety of system failures (sensors, actuators, components). The jump-process formulation (JPF) technique handles sudden shifts or jumps in the system matrices by mixing the MM method.[2] It has a manageable fixed bias size and is still suboptimal because of steady-state effects on residuals. Other methods include an algorithmic approach to FDI[7] and an expert system approach[8] among many alternatives investigated over the past years.

In many practical situations, uncertainty in the process can affect the performance of the system significantly no matter how the uncertainty is described (vagueness or ambiguity). This realization provides the motivation for a possibilistic fuzzy logic approach to FDI. This has the ability to directly describe the potential failure modes in the parameters while handling a class of nonlinear systems. To resolve the actual failure in the system parameters, a recursive parameter estimation technique is an essential component of FDI. In a soft body of consonant evidence, Zadeh's fuzzy sets or membership functions[9] can be applied to continuous decision-making processes, whereas in a distinct body of crisp evidence we can rely on Dempster-Shafer's belief or plausibility measure.[10] These approaches provide a mathematical theory of combining rules of evidence.

The objective of this paper is to develop a robust—sensitive to each failure mode and insensitive to false alarms—FDI algorithm applicable to restructurable control systems. The scheme consists of two processing units. First, fault triggering plays the role of a failure sensitive filter at the learning stage. A recursive parameter estimation technique is used for determining the direction and size of parameter changes in real time from the failure signature. Once the FDI system is alert, the histograms of parameter residues are evaluated and transformed into the corresponding fuzzy sets as decision samples at the tracking stage. In the second step, the corresponding fuzzy input vectors are applied to fuzzy decision hypercubes that undertake the identification of the failure mode utilizing the concept of possibility theory. Thus, multiple failures can be identified using the knowledge base. The combined analytical/intelligent approach presented in this paper relaxes many assumptions raised in other FDI schemes such as the conditions of a linear time-invariant system[11] and left invertibility from the failure mode to the output[12] and the requirement of a bank of Kalman filters.[13,14]

II. Failure Model via Recursive Parameter Estimation

A failure mode is defined as a one-to-one mapping relationship with a failure symptom such as "the impulse line is leaking" or "the motor inertia is very large." In fact, the failure mode is directly related to the system parameters and can be additively incorporated in a failure model. Therefore, we can easily interpret a failure mode as a physical representation of such a failure whereas a failure signature is defined as a time history of each failure mode. The actual system may be described by a state space representation in terms of discrete nonlinear vector equations of the form

$$x(t+1) = f[x(t), \theta(t), u(t)]$$

$$z(t) = h[x(t), \theta(t)] \tag{1}$$

Received Feb. 21, 1991; revision received July 26, 1991; accepted for publication Aug. 28, 1991.

*Professor, School of Electrical Engineering.
†Postdoctoral Fellow, School of Electrical Engineering.
‡Graduate Research Assistant, School of Electrical Engineering.

where $u(t) \in U$ and $z(t) \in Z$ are the scalar system input and output, respectively; $\theta(t) \in \Theta$ is the time-varying system parameter vector; $x(t) \in X$ denotes the state vector; and f and h represent the nonlinear vector fields for the state and output vectors, respectively. A nonlinear model can be obtained for a class of nonlinear systems where the corresponding equations are linear in the parameter vector $\theta(t) \in R^n$ such as

$$z(t) = \phi(t, \bar{\theta})^T\theta(t) \tag{2}$$

where $\bar{\theta}$ is the nominal solution for the parameters during the time window $J_T = [t_0, t_0 + T]$ and $\phi(t, \bar{\theta}) \in R^{p\times n}$ is the nonlinear regressor associated with $z(t - i)$ for $i = 1 \cdots r_1$ and $u(t - d - j)$ for $j = 0 \cdots r_2 - 1$ (d is an integer designating the system delay and $n = r_1 + r_2$). In the linear case, it is easy to derive the previous relation from an autoregressive moving-average (ARMA) model. The failure model can be represented by

$$\theta(t + 1) = \theta(t) + \omega(t) + \xi(t) \tag{3}$$

$$z(t) = \phi(t)^T\theta(t) + v(t) + \eta(t) \tag{4}$$

where $\omega(t) \in R^n$ and $v(t) \in R^p$ are white Gaussian parametric and measurement disturbances, respectively; $\phi(t) = \phi(t, \bar{\theta}) \in R^n$ is the regression vector; $\xi(t) \in R^n$ is the system plus actuator failure signature; and $\eta(t) \in R^p$ represents the sensor failure signature. For the initial conditions, it is required that $\theta_0 = E[\theta(0)]$ and $P_0 = \mathrm{Cov}[\theta(0) - \bar{\theta}]$. Let the parameter residue be $\tilde{\theta}(t) \in \tilde{\Theta}$, where $\tilde{\Theta}$ is the parameter residue space; $\theta(t + 1) = \theta(t) + \omega(t)$ when there is no failure ($t < t_f$),

$$\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta} \tag{5}$$

where $\hat{\theta}(t)$ is the estimate of the parameter $\theta(t)$. The parameter residues are quantized in a finite number of bins q_i with respect to the ith element of $\tilde{\theta}(t)$ to generate a histogram of the parameter residues by counting the frequency of each bin $\tilde{\theta}_k$, $f^i = \{f_k^i\} \in Q_i$ for $k = 1 \cdots q_i$ and for $i = 1 \cdots n$, and $\tilde{\theta}_k$ is the bin symbol with a total of q_i bins. Here, we call the vector $f^i \in R^{q_i}$ the failure signature histogram (FSH) of the ith system parameter. The time-window function W_T is introduced such that

$$W_T : \tilde{\Theta} \Rightarrow Q \tag{6}$$

Note that selection of the design constants is important to the performance of the hybrid FDI scheme. In the normal case, the failure signatures are

$$\xi(t) = 0, \qquad \eta(t) = 0$$

Normal Mode Operation

When $\xi(t) = 0$ and $\eta(t) = 0$, the recursive formula is obtained from the Kalman filter interpretation of Eqs. (3) and (4).

Failure Mode Operation

Let us assume that either one of the $\xi(t)$ or one of the $\eta(t)$ is nonzero and let $\xi(t) \neq 0$ for simplicity. The steady-state assumption assures that

$$\hat{\theta}(t) \Rightarrow \bar{\theta} \text{ as } t \Rightarrow \infty$$

but if the failure occurs at $t = t_f < T$,

$$\hat{\theta}(t) \Rightarrow \bar{\theta} + \sum_{\tau = t_f}^{T} \xi(\tau) \text{ as } t \Rightarrow T$$

Therefore, the parameter residue $\tilde{\theta}(t) = \hat{\theta}(t) - \bar{\theta}$ plays an important role in representing the unique failure signature by the certainty equivalence principle. This failure signature can be obtained by the windowing function W_T that stores the residue $\tilde{\theta}(t)$ in J_T into the histogram f^{θ}. The block diagram of the FDI scheme is shown in Fig. 1.

III. Failure Identification Using Fuzzy Decision Hypercubes

We present a new failure identification method using the concept of a rule base implemented by a multidimensional decision-making system called Fuzzy Decision Hypercube (FDHC). A fuzzy decision hypercube is an approximate learning and inferencing paradigm for intelligent identification and control systems.

A fuzzy variable u is a quantity with uncertainty in the universe of discourse U. For example, a fuzzy proposition F = "compressor speed is medium" may be described as

$$\mu_F = [0.2 \quad 0.7 \quad 1 \quad 0.8 \quad 0.5 \quad 0.3 \quad 0]$$

where μ_F is the membership function of u in the universe of discourse U, and the number of quantization levels is seven. The linguistic expression F = "compressor speed is very large" may be described by

$$\mu_F = [0 \quad 0 \quad 0.1 \quad 0.2 \quad 0.4 \quad 0.8 \quad 1]$$

A fuzzy set can be represented as

$$A = \{[u, \mu_F(A)] \mid u \in U\} \tag{7}$$

where μ_A is a membership function defined by

$$\mu_A : U \to [0, 1]^q \tag{8}$$

where q is the number of quantization levels.

An FDHC consists of independent fuzzy condition sets and a crisp action set. Let the number of fuzzy inputs be m, then the dimension of an FDHC is $(m + 1)$. Each input and output subset contained in an FDHC can be regarded as a fuzzy symbolic rule. It is noted that the premise part is "fuzzy" while the decision part is "crisp." In other words, the various

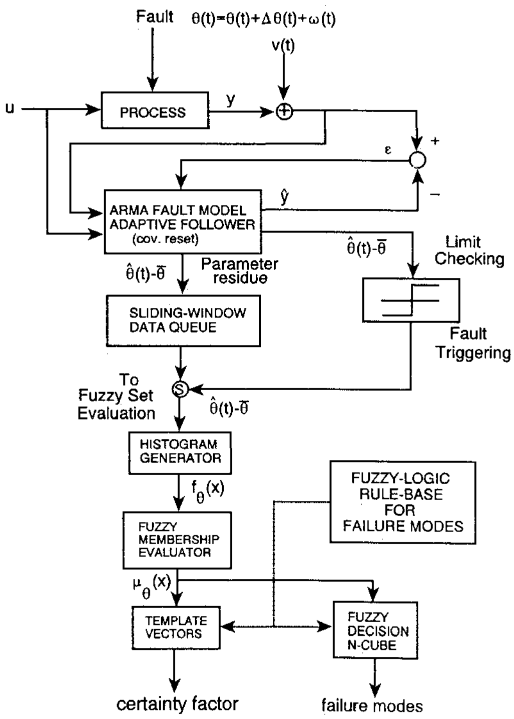

Fig. 1 Block diagram of the proposed FDI scheme.

input u are fuzzy, and the output v is crisp in an FDHC. The ith rule is

Rule i:

$$\text{if } (u_{i1} \text{ is } F_{i1}) \text{ and } \cdots \text{ and } (u_{im} \text{ is } F_{im}) \text{ then } v_i \text{ is } G_i \tag{9}$$

where F_{ij} is a fuzzy linguistic term, $A_i = A_{i1} \cap A_{i2} \cap \cdots \cap A_{im}$, A_{ij} = "u_{ij} is F_{ij}," and G_i is a crisp decision. Let us denote by

$$x_{ij} = \mu_{F_{ij}}(u_j), \qquad y_i = \mu_{G_i}(v_i)$$

where $x_{ij} \in [0, 1]^{m_j}$, $y_i \in [0, 1]^p$ and $\mu_{F_{ij}}(u_j)$ and $\mu_{G_i}(v_i)$ are the membership functions for "μ_j is F_{ij}" and "v_i is G_i," respectively. We will introduce the following notation for x_i:

$$x_i = \bigotimes_{j=1}^{m} x_{ij} \tag{10}$$

where $\otimes$ is the symbol for the orthogonal minimum operator. If we define $x_{ij}[k]$ as the kth element of the fuzzy vector x_{ij}, the previous equation can be described in detail as

$$x_i[k_1, \ldots, k_m] = x_{i1}[k_1] \otimes \cdots \otimes x_{im}[k_m] \tag{11}$$

where $k_i = 1 \cdots q_i$. Therefore, x_{ij} is a fuzzy vector, y_i is a crisp vector, and x_i is a m-dimensional hypercube. The n rules given by heuristics or the expertise of expert operators are stored in an FDHC M_n by the following recursive procedure:

$$\begin{aligned} M_n &= \bigoplus_{i=1}^{n} (x_i \otimes y_i) \\ &= M_{n-1} \oplus (x_n \otimes u_n) \end{aligned} \tag{12}$$

where $M_n \in [0, 1]^{q_1} \times \cdots \times [0, 1]^{q_m} \times [0, 1]^p$; $\oplus$ and $\otimes$ are the maximum and the orthogonal minimum operators, respectively.

Let us assume that the M_n hypercube has stored n rules of the form described in Eq. (9). If the input to the hypercube is now a perturbed sequence of vectors $\tilde{x}_1, \ldots, \tilde{x}_m$, the question arises as to how we infer the corresponding output vector $\tilde{y}$? To achieve this reasoning step, we call upon the compositional rule of inference or generalized "modus ponens." In a fuzzy sense, this is described as follows:

Fuzzy Modus Ponens:
- Premise: $(\tilde{x}_1)$ and $\cdots$ and $(\tilde{x}_m)$
- Implication: If (x_1^k) and $\cdots$ and (x_m^k), then (y^k)
- Consequence: $(\tilde{y})$

Now let us consider the FSH for *only one* parameter and let the number of the associated failure mode be ℓ. How is the FDHC used as an approximate reasoning or association tool? As we suggested earlier, we begin with a histogram in which conditional information is submerged. Let the normalized vector histogram be f and its kth quantization bin $f[k]$, then

$$f = \{f[1], \ldots, f[q_m]\} \tag{13}$$

Given an FSH f^θ, there exists a transformation π from the probabilities to the corresponding fuzzy sets x_θ that is based on possibility theory.[9] A numerical evaluation is given in Refs. 15 and 16 via the formula

$$\pi(\{\theta_k\}) = \sum_{j=1}^{q} \min\{f^\theta[k], f^\theta[j]\} \tag{14}$$

where $f^\theta[k]$ is sorted according to the frequency of each bin θ_k with the condition $\Sigma_{k=1}^{q} f^\theta[k] = 1$. The possibility transformation π provides the maximum probability of the parameter residue by a mapping

$$\pi : Q \Rightarrow [0, 1] \tag{15}$$

where we define $\tilde{x}_\theta[k] = \pi(\{\theta_k\})$. The fuzzy vector $\tilde{x}_\theta$ is stored as a fuzzy input vector to the FDHC. Thus, each failure mode can be represented uniquely by a failure signature fuzzy set (FSF). We also define a priori failure mode fuzzy sets (FMF) that correspond to each crisp failure mode. The procedure requires two transformations:

$$\tilde{\Theta} \xrightarrow{W_T} Q_{(\mathrm{FSH})} \xrightarrow{\pi} [0, 1]_{(\mathrm{FSF})} \tag{16}$$

The first one transforms the residual information into a histogram in Eq. (6), i.e., it carries the residue data from the parameter space to a probability space through the windowing function W_T. The histogram is transformed next to a fuzzy set through π as explained earlier in Eq. (15).

The FDHC is constructed along each rule on the basis of a priori information about the specific FSF and FMF. In general, each rule is described by a fuzzy implication whose premise consists of conjunctive fuzzy sets.

Now, using the compositional rule of inference, the output fuzzy vector for each parameter failure can be expressed by

$$\tilde{y}[i] = \bigoplus_{k} \{M_n[i, k] \otimes \tilde{x}_\theta[k]\} \tag{17}$$

where $\tilde{x}_\theta = \{\tilde{x}_\theta[k], k = 1, \ldots, q\}$ is the stored fuzzy set from the actual failure signature that will be applied to the input of the FDHC, and $\tilde{y} = \{\tilde{y}[i], i = 1, \ldots, \ell\}$ is the output of the FDHC. Therefore, the resultant FMF output i^* excited by the FSF vector $\hat{x}_\theta$ is obtained by defuzzification,

$$i^* = \arg\max_i \tilde{y}[i] \tag{18}$$

When we have multiple $\tilde{x}_\theta$, $\tilde{x}_\theta$ and M_n of Eq. (17) will be expanded into hypercubes of dimension m and $(m + 1)$, respectively. How is the FDHC used as an approximate reasoning or association tool? As we suggested earlier, we begin with a histogram in which conditional information is submerged. Let the normalized vector histogram of the ith fuzzy set be f_i and its kth quantization bin be $f_i[k]$, then

$$f_i = \{f_i[1], \ldots, f_i[q_m]\} \tag{19}$$

The fuzzy sets are evaluated based on the equation given by Dubois and Prade[15] and Yager,[16]

$$\tilde{x}_i[j] = \sum_{k=1}^{q_i} \{f_i[j] \otimes f_i[k]\} \tag{20}$$

and the fuzzy reasoning mechanism is

$$y = M_n \circ x \tag{21}$$

where x is defined as

$$x = x[k_1, \ldots, k_m] = \otimes_{j=1}^{m} x_{0j}[k_j] \tag{22}$$

and $\circ$ is the max-min operator. Therefore, a crisp decision is made according to the algorithm

$$v^* = \arg\max_v \mu_{G_i}(v) = \arg\max_v y \tag{23}$$

where v^* is the resultant mode.

IV. Degree of Certainty via Dempster-Shafer Theory

In parallel with the fuzzy decision hypercube (FDHC), the Dempster-Shafer (DS) module is used as the last stage of the FDI algorithm to provide a quantitative measure of the confidence level in the decision-making process. The DS theory is a theory of evidence because it deals with weights of evidence and numerical degrees of support based on evidence. At the same time, it is also a theory of probable reasoning because it focuses on the fundamental operation of probable reasoning, namely, the combination of evidence. Since the DS module shares the same rule base with the FDHC, it produces consistent results with regard to the particular failure mode detected. Moreover, the DS module provides a reliability index called "degree of certainty" by resolving conflicting information and combining effectively the available evidence as an aid toward the final decision. According to the DS theory, an expression for the measure of belief and the combination of evidence require the evaluation of the basic assignment. The latter is interpreted as the measure of belief that is committed exactly by its focal element and is obtained by the following step-wise procedure:

1) Find a likelihood measure that is defined by

$$L_i = \sum_j \min(T_{ij}, D_j)$$

where T_{ij} is the jth value of template vector T_i and D_j is the count of the jth interval.

2) The possibility distribution is calculated after normalizing and reordering the L_i.

3) The basic assignments m are computed, finally, from the possibility distribution.

Example 1

Let π_i be the normalized and reordered L_i such that $\pi_1 = 1$ and $\pi_i \geq \pi_{i+1}$, then $m(A_i) = \pi_i - \pi_{i+1}$, where A_i includes the first ith largest mode. For example, if $L_1 = 0.4$, $L_2 = 1$, and $L_3 = 0.6$, then $A_1 = x_2$, $A_2 = x_2x_3$, and $A_3 = x_2x_3x_1$, and $\pi_1 = 1$, $\pi_2 = 0.6$, and $\pi_3 = 0.4$. Thus, $m(x_2) = 0.4$, $m(x_2x_3) = 0.2$, and $m(x_2x_3x_1) = 0.4$

If we have more than one estimate as a clue for identifying a failure, the same number of sets of basic assignments is available. It is not unusual that several symptoms (failure signatures) are related to a specific failure mode. We introduce the concept of a pseudofailure mode to reduce the computational burden. This is accomplished by using a failure signature as a focal element rather than a failure mode. Assume that p_i indicates the number of failure signatures for the ith parameter, which is less than the number of failure modes M, the number of multiplications is reduced then to $p_1 \times p_2, \ldots, \times p_n$ from M^n. In this case, pseudofailure modes appear as an intermediate stage in the decision-making process and the difficulty with associating failure signatures to failure modes is resolved. The following steps are required to obtain the basic assignment using the intermediate stage:

1) Find a likelihood measure L_i where the range of i depends on the number of failure signatures for each parameter.

2) Compute the basic assignments m from the L_i and replace their focal elements with the pseudofailure modes.

3) Combine the basic assignments for each parameter.

4) Change the focal elements of the resultant basic assignments with failure modes.

Example 2

To illustrate the notions proposed earlier, we consider three measurable parameters in the case of an axial flow compressor (the example is further discussed in the following section of the paper). Let us assume that the following observations are available:

Compressor speed (B): $L_S = 0.4$, $L_M = 0.8$, $L_L = 0.2$, $L_{VL} = 0.2$

Throttle nozzle area (α): $L_S = 0.6$, $L_M = 0.3$, $L_L = 0.15$

Pressure difference (ΔP): $L_{NO} = 0.5$, $L_{SO} = 0.2$, $L_{VO} = 0.0$

After normalizing the likelihood measure, the basic assignments whose focal elements are the pseudofailure modes (PSM) can be obtained. The basic assignments for the throttle nozzle area m_α are

m_α(PSM25, . . . , PSM36) = 0.5

m_α(PSM13, . . . , PSM25, . . . , PSM36) = 0.25

m_α(PSM1, . . . , PSM13, . . . , PSM25, . . . , PSM36) = 0.25.

Combining the three sets of basic assignments (m_B, m_α, $m_{\Delta P}$) produces

m(PSM26) = 0.15, m(PSM26, PSM30) = 0.1, m(PSM26, PSM14) = 0.075 . . .

Then the pseudofailure mode is replaced with the corresponding failure mode. This yields the following:

m(FM4) = 0.15 + 0.1 + 0.075 + 0.05 = 0.375

m(FM4, FM3) = 0.075 + 0.05 + 0.0375 + 0.025 = 0.1875

⋮

m(FM1, FM2, FM3, FM4, FM5, FM7) = 0.025.

The DS module is employed with a combination of sets of basic assignments and pseudofailure modes providing as an output the identified failure modes and their associated degree of certainty. A set of basic assignments whose focal elements are failure modes results after combining all sets. Here we use the concept of degree of certainty that is defined as $m(X) - Bel(\overline{X})$, where $Bel(X) = \Sigma_{Y \subset X} m(Y)$. If the degree of certainty is one, it implies that the decision about X is absolutely correct. If $m(X)$ is equal to $Bel(\overline{X})$, that is, the belief and disbelief in the occurrence of X are equally weighted, then it is plausible to conclude that the decision is quite uncertain. Therefore, $m(X) - Bel(\overline{X})$ is a proper choice for the degree of certainty. In the previous example, X is FM4 and its degree of certainty is 0.375, since Bel(FM1, FM2, FM3, FM5, FM6, FM7) equals zero.

V. Application of FDI to the Axial Flow Compressor Problem

The FDI algorithmic developments, described in the previous sections of the paper, were applied to a major system of a turbojet engine. The surge and rotating stall post instability behavior of axial flow compressors has received considerable attention over the past years.[17,18] Aerodynamic instabilities of the compressor can severely limit its operating envelope. When the compressor is near its maximum achievable pressure rise, a moderate disturbance can result in instability or even loss of the nominal operating point. The compression system can then assume two types of dynamic behavior: the first, a large amplitude, low frequency oscillation, known as surge, in which the compressor experiences a series of rapid flow reversals and recovery; the second, known as rotating stall, is characterized by very inefficient engine operation at constant mass flow and pressure rise. Recovery from a stall condition usually necessitates engine shutdown and restart. We pose the following question: Is it possible to identify an instability condition (and, preferably, an *impending* instability) by monitoring key compressor dynamic variables? If the answer to this question is affirmative, then can we use these data in the form of failure signatures to detect and identify specific instability conditions by assigning to them the role of failure modes?

The technical approach we pursued to address this problem includes the following tasks: 1) nonlinear modeling of axial flow compressor dynamics, 2) identification of failure modes as instability conditions, 3) definition of measurable failure signatures, and 4) application of the hybrid FDI algorithm, assessment, and evaluation of simulation results.

Compressor Dynamics

We have adopted Greitzer's fourth-order lumped parameter model[17] augmented by Nett's B-forcing term[19] and an α-forcing term introduced by us to account for the dynamics of throttle area variations. The full sixth-order one-dimensional model (in nondimensional form) is given by

Compressor:

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{\bar{\tau}}[\bar{C}_{ss}(\dot{\bar{m}}_c) - \bar{C}] - 2\frac{\bar{C}}{B}\frac{\partial B}{\partial \bar{t}} \tag{24}$$

Rule #	B	α	ΔPc	Failure Mode
1	S	L	NO	1 NOP
2	M			1 NOP
3	L			1 NOP
4	VL			1 NOP
5	S	L	SO	2 POP
6	M			1 NOP
7	L			1 NOP
8	VL			2 POP
9	S	L	FO	2 POP
10	M			1 NOP
11	L			1 NOP
12	VL			2 POP
13	S	M	NO	3 IST
14	M			3 IST
15	L			2 POP
16	VL			2 POP
17	S	M	SO	3 IST
18	M			3 IST
19	L			5 ISG
20	VL			5 ISG
21	S	M	FO	3 IST
22	M			3 IST
23	L			5 ISG
24	VL			5 ISG
25	S	S	NO	4 RST
26	M			4 RST
27	L			5 ISG
28	VL			5 ISG
29	S	S	SO	4 RST
30	M			4 RST
31	L			7 DSG
32	VL			7 DSG
33	S	S	FO	4 RST
34	M			4 RST
35	L			6 ASG
36	VL			7 DSG

Note :
1. NOP : Normal Operation
2. POP : Potential Unstable
3. IST : Impending Surge
4. RST : Rotating Stall
5. ISG : Impending Surge
6. ASG : Abrupt Surge
7. DSG : Deep Surge

S : Small
M : Medium
L : Large
VL : Very Large
NO : No Oscillation
SO : Slow Oscillation
FO : Fast Oscillation

Fig. 2 Fuzzy rule base for compressor example.

Compressor duct:

$$\frac{\partial \dot{\bar{m}}_c}{\partial \bar{t}} = B(\bar{C} - \Delta\bar{P}) - \frac{\dot{\bar{m}}_c}{B} \tag{25}$$

Throttle duct:

$$\frac{\partial \dot{\bar{m}}_t}{\partial \bar{t}} = \frac{B}{G}(\Delta\bar{P} - \bar{F}) - \frac{\dot{\bar{m}}_t}{B}\frac{\partial B}{\partial \bar{t}}, \quad G(t) = \frac{L_T A_c}{L_c A_T(t)} \tag{26}$$

Plenum:

$$\frac{\partial \Delta\bar{P}}{\partial \bar{t}} = \frac{1}{B}(\dot{\bar{m}}_c - \dot{\bar{m}}_t) - 2\frac{\Delta\bar{P}}{B}\frac{\partial B}{\partial \bar{t}} \tag{27}$$

Rotor speed: The so-called B parameter whose value is proportional to rotor speed $U(t)$

$$\frac{\partial B}{\partial \bar{t}} = \frac{1}{\tau_B}(B_\infty - B), \quad B(t) \overset{\text{def}}{=} \frac{U(t)}{2\omega L_c} \tag{28}$$

Throttle area dynamics:

$$\frac{\partial \alpha}{\partial \bar{t}} = \frac{1}{\tau_\alpha}(\alpha_\infty - \alpha), \quad \alpha(t) = \frac{A_T(t)}{A_c} \tag{29}$$

where τ_B and τ_α are the time constants of the compressor rotor and throttle, respectively. The algebraic equations de-

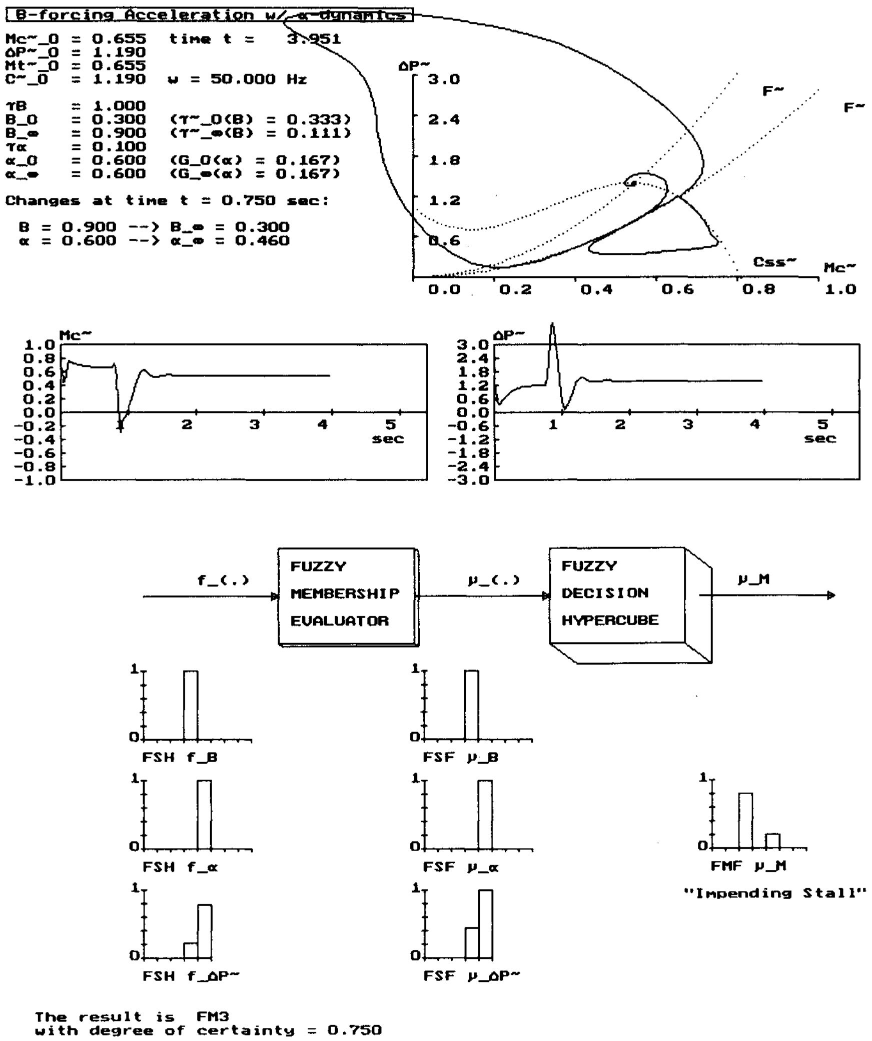

Fig. 3 Simulation results for an impending stall.

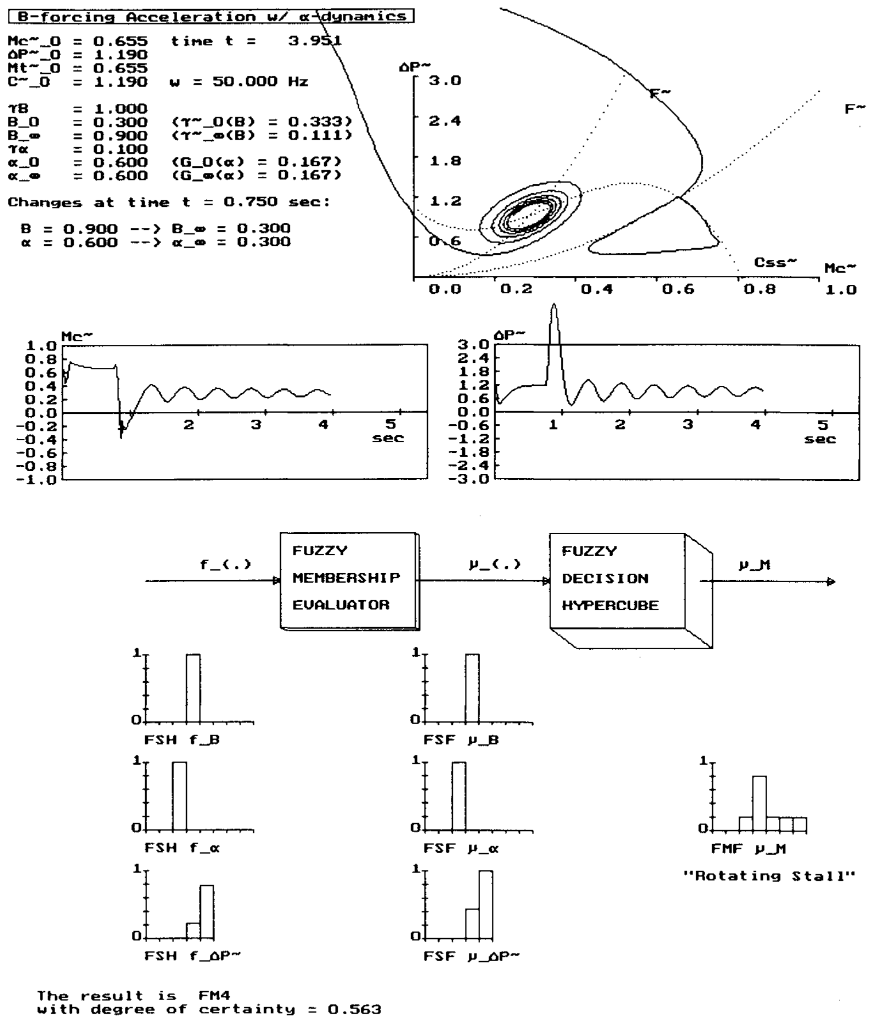

Fig. 4 Simulation results for a rotating stall.

scribe the nonlinear compressor performance and throttle characteristics. They are approximated by

$$\tilde{C}_{ss} = -22.35\dot{m}_c^3 + 22.86\dot{m}_c^2 - 5.157\dot{m}_c + 1.04 \quad (30)$$

$$\tilde{F} = \left(\frac{\dot{m}_t}{\alpha}\right)^2 \quad (31)$$

Failure Signatures

Three measured parameters define corresponding failure signatures for FDI purposes. They are the following:

1) FS1 $B(t)$—through a direct measurement of rotor rotational speed (rpm). Small values of B are indicative of a stall condition whereas the compressor tends to a surge instability for large values of this parameter [normal value of $B(t) = 0.7 \sim 0.8$].

2) FS2 $\alpha(t)$—through a direct measurement of the throttle area (normally $\alpha > 0.6$).

3) FS3 $\Delta\tilde{P}$—through a direct measurement of the differential pressure between the compressor and atmospheric conditions. An electric circuit basically consisting of a zero-crossing counter provides information about the frequency of the $\Delta\tilde{P}$ oscillations.

Failure Modes

The rationale for this particular choice of failure signatures and failure modes stems from the analysis of the compressor dynamic behavior at or near stall and surge conditions. The following failure modes are identified:

1) FM1 normal operation = NOP. The designation is obvious but must be capable of avoiding false alarms.

2) FM2 potentially unstable operation = POP. When the compressor performance table (CPT) and the throttle characteristics (TC) are both within their normal range but the value of B is either too large or too small indicating that the compressor is vulnerable to entering an unstable operating region.

3) FM3 impending stall = IST. IST is signified by the CPT and TC operating at their corresponding peak points; small disturbances about the operating point due to throttle behavioral uncertainty may cause the system to move toward a rotating stall condition.

4) FM4 rotating stall = RST. The compressor exhibits an RST mode when the TC has fallen below the peak operating point.

5) FM5 impending surge = ISG. A very large value of B in the absence of $\Delta\tilde{P}_c$ oscillations or an uncertain TC position in combination with a large value of B will cause the compressor to exhibit eventually a surge instability. An ISG condition is not accompanied by pressure oscillations.

6) FM6 abrupt surge = ASG. Large values of B in combination with a high frequency oscillatory behavior of $\Delta\tilde{P}_c$ and uncertain α(TC) values signal the presence of an ASG.

7) FM7 deep surge = DSG. DSG condition is evidenced by very large values of B accompanied by slow oscillations and small values of α(TC).

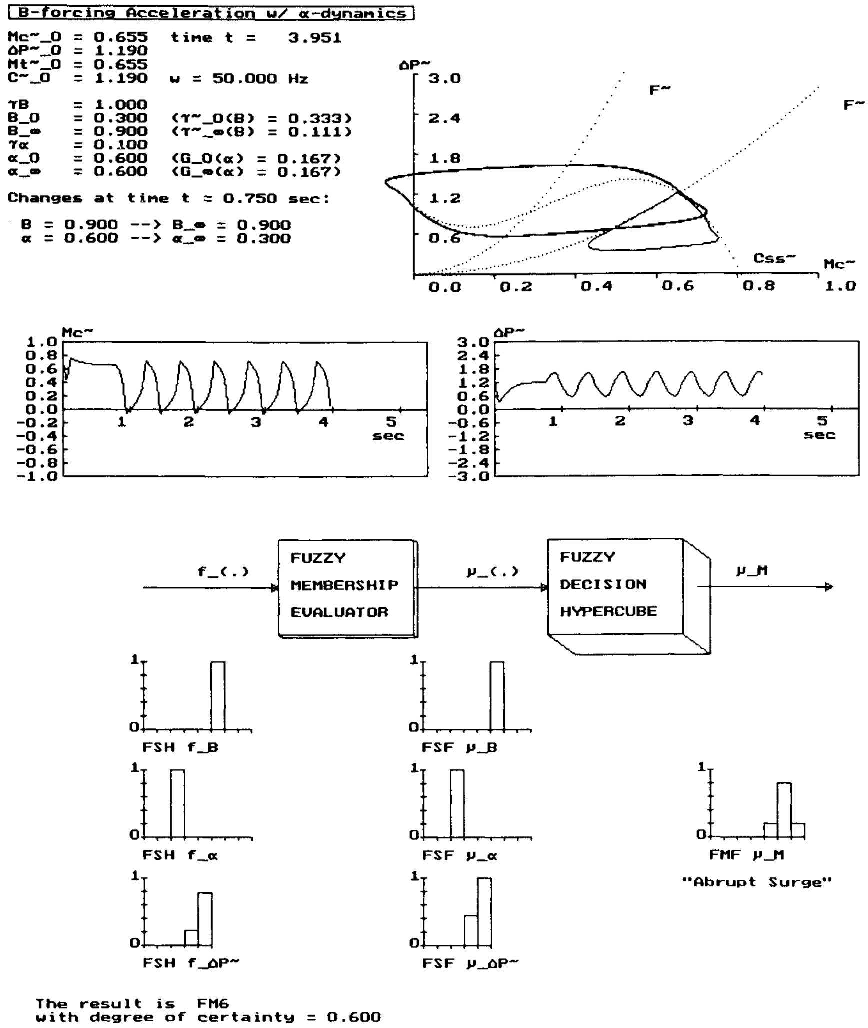

Fig. 5 Simulation results for an abrupt surge.

For the compressor case, the linguistic terms for the failure signatures and failure modes are defined as follows.

FS1 B—Compressor speed:

$$L(B) = \{\text{Small, Medium, Large, Very Large}\} = \{S, M, L, VL\}$$

FS2 α—Throttle nozzle area:

$$L(\alpha) = \{\text{Small, Medium, Large}\} = \{S, M, L\}$$

FS3 ΔP—Pressure difference:

$$L(\Delta P) = \{\text{No Osc., Small Osc., Very Osc.}\} = \{NO, SO, VO\}$$

$$L(\text{FM}) = \{\text{NOP, POP, IST, RST, ISG, ASG, DSG}\}$$

An exhaustive enumeration of all possible FS and FM combinations leads to a total of 36 rules. With prescribed membership functions derived primarily from simulation studies, the rule base is shown in Fig. 2.

Each rule is of the form

$$\text{If } B \text{ is } (\cdots) \text{ and } \alpha \text{ is } (\cdots) \text{ and } \Delta P \text{ is } (\cdots), \text{ then FM is } (\cdots)$$

i.e., the fuzzy decision hypercube (FDHC) uses fuzzy set inputs and produces a crisp output that corresponds to the identified failure mode.

Dempster-Shafer theory provides a suitable framework for ignorance management. A likelihood distribution is derived first from the failure signature possibilistic distribution. Basic assignments are evaluated next from the likelihood values, after the latter have been reordered and normalized. The evidence is finally combined and the degree of certainty is computed via

$$\text{Degree of Certainty} = m(X) - Bel(\overline{X})$$

Figures 3, 4, and 5 show typical computer simulation results for an impending stall, a rotating stall, and an abrupt surge instability condition, respectively. Extensive simulation runs under various FS scenarios have produced similar results. Since the fuzzy decision hypercube and the template vectors use the same fuzzy failure signature distributions, the results are both consistent and complementary. That is, the FDHC decides on the particular FM while managing system uncertainty and the template vector algorithm corroborates this decision and assigns a degree of certainty to it that accounts for incomplete or conflicting evidence in the failure signature information. The combined algorithm thus proves to be an efficient tool for decision making in the presence of uncertainty and ignorance—a situation typified by conditions in an axial flow compressor.

VI. Conclusions

Failure detectability and identifiability concepts are studied in terms of recursive parameter estimation as well as fuzzy decision hypercubes. We present a hybrid analytical/intelligent approach to fault detection and identification for restructurable control systems that guarantees maximum sensitivity to failure modes and minimum sensitivity to false alarms. The procedure consists of a simple limit checking to detect the failure, a covariance-resetting recursive estimation to evaluate the failure signature, use of fuzzy decision hypercube to identify the failure mode, and Dempster-Shafer theory to assign a reliability index to the decision process. Possibility theory provides a reasoning tool for the case where a large grained uncertainty in the data base and no a priori information, which is absolutely necessary in Bayesian theory, is available. The main advantage of possibility theory, compared with probabilistic methods, is computational speed. This objective is achieved since the basic calculations in possibility theory are carried out by using maximum and/or minimum operations.

In the axial flow compressor example, the failure modes (instabilities) are defined as the combined effects of the system parameters B and α. The decision task is based on a fuzzy representation of these measurable system parameters and the monitored plenum pressure ΔP_c. Dempster-Shafer theory is introduced to combine the three failure signatures and produce a crisp failure mode. Conventional fault detection and identification techniques typically involve increased dimensionality and algorithm complexity due to the inherent coupled treatment of failure signatures. Thus, unique features of the proposed approach refer to the efficient treatment of uncertain and incomplete information. The approach eases considerably the computational burden and allows for an on-line real-time implementation of the fault detection and identification routines. The fuzzy decision hypercubes can be realized at the expense of some hardware complexity offering the possibility for viable real-time applications of the fault detection and identification algorithms. The Dempster-Shafer theoretic technique not only provides a complementary decision-making tool but also can be used as a measure of sensitivity through the degree of certainty provision.

Acknowledgment

This research was sponsored by the Office of Naval Research, Applied Research and Technology Directorate, under Contract N00014-89-5-3113.

References

[1]Chow, E. Y., and Willsky, A. S., "Analytical Redundancy and the Design of Robust Failure Detection Systems," *IEEE Transactions on Automatic Control*, Vol. AC-29, No. 7, July 1984, pp. 689–691.

[2]Willsky, A. S., "A Survey of Design Methods for Failure Detection in Dynamic Systems," *Automatica*, Vol. 12, No. 6, Nov. 1976, pp. 601–611.

[3]Isermann, R., "Process Fault Detection Based on Modelling and Estimation Methods—A Survey," *Automatica*, Vol. 20, No. 4, July 1984, pp. 387–404.

[4]Deckert, J. C., Desai, M. N., Deyst, J. J., and Willsky, A. S., "F-8 dfbw Sensor Failure Identification Using Analytic Redundancy," *IEEE Transactions on Automatic Control*, Vol. AC-22, No. 5, Oct. 1977, pp. 795–803.

[5]Jones, H. L., "Failure Detection in Linear Systems," Ph.D. Thesis, Department of Aeronautics and Astronautics, Massachusetts Inst. of Technology, Cambridge, MA, Aug. 1973.

[6]Kailath, T., *Linear Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1980.

[7]Kim, J. H., and Bien, Z., "An Algorithmic Approach to Fault Diagnosis in Linear Systems," *IEEE Transactions on Industrial Electronics*, Vol. 36, No. 3, Aug. 1989, pp. 313–320.

[8]Handelman, D. A., and Stengel, R. F., "Combining Quantitative and Qualitative Reasoning in Aircraft Failure Diagnosis," *Proceedings of the 1985 AIAA Guidance, Navigation, and Control Conference*, AIAA, New York, 1985, pp. 375–384; AIAA Paper 85-1905, Aug. 1985.

[9]Zadeh, L. A., "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," *IEEE Transactions on System Man Cybernetics*, Vol. SMC-3, No. 1, Jan. 1973, pp. 28–44.

[10]Shafer, G., *A Mathematical Theory of Evidence*, Princeton Univ. Press, Princeton, NJ, 1976.

[11]Willsky, A. S., and Jones, H. L., "A Generalized Likelihood Ratio Approach to the Detection and Estimation of Jumps in Linear Systems," *IEEE Transactions on Automatic Control*, Vol. AC-21, No. 1, Feb. 1976, pp. 108–112.

[12]Massoumnia, M. A., Verghese, G. C., and Willsky, A. S., "Failure Detection and Identification," *IEEE Transactions on Automatic Control*, Vol. AC-34, No. 3, March 1989, pp. 316–321.

[13]Chang, C. B., and Athans, M., "State Estimation for Discrete Systems with Switching Parameters," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-14, No. 3, May 1978, pp. 418–425.

[14]Tugnait, J. K., "Detection and Estimation for Abruptly Changing Systems," *Automatica*, Vol. 18, No. 5, 1982, pp. 607–615.

[15]Dubois, D., and Prade, H., "On Several Representations of an Uncertain Body of Evidence," *Fuzzy Information and Decision Processes*, edited by M. M. Gupta and E. Sanchez, North-Holland, Amsterdam, 1982, pp. 167–181.

[16]Yager, R. R., "Level Sets for Membership Evaluation of Fuzzy Subsets," *Fuzzy Set and Possibility Theory: Recent Developments*, edited by R. R. Yager, Pergamon, Oxford, England, UK, 1982, pp. 90–97.

[17]Greitzer, E. M., "Surge and Rotating Stall in Axial Flow Compressors, Part i, ii," *ASME Journal of Engineering for Power*, Vol. 98, No. 2, April 1976, pp. 190–217.

[18]Abed, E. H., Houpt, P. K., and Hosny, W. M., "Bifurcation Analysis of Surge and Rotating Stall in Axial Flow Compressors," *Proceedings of the 28th American Control Conference*, Institute of Electrical and Electronics Engineers, San Diego, CA, June 1990, pp. 2239–2246.

[19]Nett, C., "Toward a New Generation of High-Performance Aircraft Gas Turbine Engine Controls: A Systems-Oriented Research Program," *Second Review Meeting of the ONR Accelerated Research Initiative on Active Control*, California Inst. of Technology, Pasadena, CA, April 1990.